Problems on Binomial Distribution :

1. Find the Binomial distribution for which the mean is 4 and variance is 3.

Solution:

For the Binomial distribution

mean $= np$; Variance $= npq$

Given $np = 4$; $npq = 3$

$\therefore \quad 4q = 3$

$$\boxed{q = \frac{3}{4}}$$

wkt $p + q = 1$

$\Rightarrow p = 1 - q$

$p = 1 - \frac{3}{4} = \frac{1}{4}$; $\boxed{p = \frac{1}{4}}$

when $p = \frac{1}{4}$; $np = 4$

$\Rightarrow n\left(\frac{1}{4}\right) = 4$

$\Rightarrow \boxed{n = 16}$

$\therefore$ The Binomial distribution is $P\{X = x\} = nC_x \, p^x \, q^{n-x}$

$$P\{X = x\} = 16C_x \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{16-x}$$

2. In a large consignment of electric bulbs 10 % are defective, a random sample of 20 is taken for inspection. Find the probability that (i) all are good bulbs (ii) atmost there are 3 defective bulbs (iii) Exactly there are 3 defective bulbs.

Solution :

Given $p = 10\% = \frac{10}{100}, = 0.1$

$n = 20$

$p = 0.1, \quad q = 1 - p = 1 - 0.1 = 0.9$

$P\{X = x\} = nC_x \, p^x q^{n-x}$

$= 20C_x (0.1)^x (0.9)^{20-x}$

(i) $P\{\text{all are good}\} = P\{x = 0\}$

$= 20C_0 (0.1)^0 (0.9)^{20}$

$= (0.9)^{20}$

$$\boxed{P\{\text{all are good}\} = 0.12}$$

(ii) $P\{\text{atmost 3 defective}\} = P\{X \le 3\}$

$= P\{X=0\} + P\{X=1\} + P\{X=2\} + P\{X=3\}$

$= 20C_0 (0.1)^0 (0.9)^{20} + 20C_1 (0.1)^1 (0.9)^{19} +$
$20C_2 (0.1)^2 (0.9)^{18} + 20C_3 (0.1)^3 (0.9)^{17}$

$P\{X \le 3\} = 0.12 + 0.27 + 0.285 + 0.19$

$= 0.865$

(iii) $P\{\text{exactly 3 are defective}\} = P\{X=3\}$

$= 20C_3 (0.1)^3 (0.9)^{17}$

$= 0.19$

3. A machine manufacturing screws is known to produce 5% defective. In a random sample of 15 screws, what is the probability that there are (i) exactly 3 defectives (ii) not more than 3 defectives.

Solution

probability of defective screws $= 5\%$

$p = \frac{5}{100} = 0.05$, Given $n = 15$

$q = 1 - p = 1 - 0.05 = 0.95$

By BD $P\{X = x\} = nC_x p^x q^{n-x} = 15C_x (0.05)^x (0.95)^{15-x}$

(i) $P\{\text{exactly 3 defectives}\} = P\{X = 3\}$

$= 15C_3 (0.05)^3 (0.95)^{12}$

$= 0.03$

$P\{\text{not more than } 3 \text{ defectives}\} = P\{X < 3\}$

$= P\{X=0\} + P\{X=1\} + P\{X=2\}$

$= 15C_0 (0.05)^0 (0.95)^{15} + 15C_1 (0.05)^1 (0.95)^{14} +$

$15C_2 (0.05)^2 (0.95)^{13}$

$= 0.463 + 0.366 + 0.135$

$= 0.964$

4 ~~18~~ Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) Atleast 1 boy (iii) Atmost 2 girls (iv) children of both genders. Assume equal probabilities for boys and girls.

Soln:

Given $n = 4$, $N = 800$

$p = \frac{1}{2}$, $q = \frac{1}{2}$ ($\because$ equal probability for boys and girls)

By Binomial distribution

$P\{X = x\} = nC_x \, p^x \, q^{n-x}$

$= 4C_x \left(\frac{1}{2}\right)^x \left(\frac{1}{2}\right)^{4-x}$

$P\{X = x\} = 4C_x \left(\frac{1}{2}\right)^4$

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out of 800 families

$P\{X=x\} = N\, P\{X=x\}$

$= (500)\; 4C_x \left(\frac{1}{2}\right)^4$

(i) $P\{$2 boys and 2 girls$\}$

$= P\{X=2\} = 4C_2\left(\frac{1}{2}\right)^4$

$= 0.375$

For out of 800 families $P\{X=2\} = 800 \times 0.375$

$= 300$ families

(ii) $P\{$atleast one boy$\} = P\{X \geq 1\} = 1 - P\{X<1\}$

$= 1 - (P\{X=0\})$

$= 1 - \left(4C_0 \left(\frac{1}{2}\right)^4\right)$

$= 1 - 0.0625$

$= 0.9375$

For out of 800 families $P\{X \geq 1\} = 800 \times 0.9375$

$= 750$ families

(iii) $P\{\text{atmost 2 girls}\} = P\{X \leq 2\}$

$$= P\{X=0\} + P\{X=1\} + P\{X=2\}$$

$$= 4C_0\left(\tfrac{1}{2}\right)^4 + 4C_1\left(\tfrac{1}{2}\right)^4 + 4C_2\left(\tfrac{1}{2}\right)^4$$

$$= 0.6875$$

out of 800 families

$P\{X \leq 2\} = 800 \times 0.6875 = 550$ families

(iv) $P\{\text{children of both genders}\}$

$$= 1 - P\{\text{Same genders}\}$$

$$= 1 - [P\{X=0\} + P\{X=4\}]$$

$$= 1 - \left[4C_0\left(\tfrac{1}{2}\right)^4 + 4C_4\left(\tfrac{1}{2}\right)^4\right]$$

$$= 1 - 0.125$$

$$= 0.875$$

Out of 800 families $P\{\text{children of both genders}\}$

$$= 0.875 \times 800$$

$$= 700 \text{ families.}$$

PROBLEMS ON POISSON DISTRIBUTION :

1. The number of typing mistakes that a typist makes on a given page has a poisson distribution with a mean of 3 mistakes. What is the probability that she makes (i) Exactly 7 mistakes (ii) Fewer than 4 mistakes (iii) No mistakes on a given Page.

Soln:

WKT the mean of the poisson distribution is λ.

Given $\lambda = 3$

The Pdf of poisson disribution is

$$P\{X=x\} = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-3}3^{x}}{x!}$$

(i) P{ she makes exactly 7 mistakes}

$$= P\{X=7\}$$

$$= \frac{e^{-3}3^{7}}{7!} = 0.02$$

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(ii) P{ She makes fewer than 4 mistakes } $= P\{X<4\}$

$$= P\{X=0\}+P\{X=1\}+P\{X=2\}+P\{X=3\}$$

$$= \frac{e^{-3}3^0}{0!}+\frac{e^{-3}3^1}{1!}+\frac{e^{-3}3^2}{2!}+\frac{e^{-3}3^3}{3!}$$

$$= e^{-3}[1+3+4.5+4.5]$$

$$= 0.647$$

(iii) P{ No mistakes on a given Page } $= P\{X=0\}$

$$= \frac{e^{-3}3^0}{0!} = 0.0479$$

2. In a certain factory producing razor blades, there is a small chance $\frac{1}{500}$ for any blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing

(i) no defective blade (ii) atleast 1 defective blade

(iii) atmost 1 defective blade in a consignment of 10,000 packets.

Soln:

P{ razor blade to be defective} = $\frac{1}{500}$ = P

Given $n = 10$

wkt $\lambda = nP$

$$\lambda = 10 \times \frac{1}{500} = \frac{1}{50} = 0.02$$

For Poisson distribution the pdf / pmf is

$$P\{X=x\} = \frac{e^{-\lambda}\lambda^{x}}{x!} = \frac{e^{-0.02}(0.02)^{x}}{x!}$$

(i) P{ no defective blade} = $P\{X=0\}$

$$= \frac{e^{-0.02}(0.02)^{0}}{0!}$$

$$= e^{-0.02} = 0.980$$

In a consignment of 10,000 packets $P\{X=0\} = 10000 \times 0.980$

= 9,800 packets.

(ii) P{atleast 1 defective blade} = $P\{X \geq 1\}$

$$= 1 - P\{X < 1\}$$

$$= 1 - P\{X=0\}$$

$$= 1 - 0.980 = 0.02$$

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In a consignment of 10,000 packets, P{atleast 1 defective blade} = 10,000 × 0.02 = 200 packets

(viii) P{atmost 1 defective blade} = P{x ≤ 1}

$= P\{x=0\} + P\{x=1\}$

$$= \frac{e^{-0.02}(0.02)^0}{0!} + \frac{e^{-0.02}(0.02)^1}{1!}$$

$= 0.980 + 0.0196 \simeq 1.0$

In a consignment of 10,000 packets,

P{atmost 1 defective blade} = 10,000 × 1.0

= 10,000 packets.

3. In a book of 520 pages, 390 typographical errors occur. Assuming Poisson law for the number of errors per page.

Soln:

$$\lambda = \frac{390}{520} = 0.75$$

$\lambda = 0.75$

$$P\{x=x\} = \frac{e^{-\lambda}\lambda^x}{x!}$$

P{ a page will contain no error} = P{ x = 0 }

$$= \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$= \frac{e^{-0.75} (0.75)^{0}}{0!}$$

$$= 0.472$$

Out of 5 Pages

P{ no error } = $(0.472)^5$ = 0.02